U6 Chapter 4

Binomial Expansion

Chapter Overview

1. Binomial Series Recap

2. Binomial Expansion for negative/fractional powers

3. Constant is not 1: $(a+b)^n$

4. Using Partial Fractions

4 **Sequences and series**	4.1	**Understand and use the binomial expansion of $(a+bx)^n$ for positive integer n; the notations $n!$ and ^nC_r link to binomial probabilities.**	**Use of Pascal's triangle.** **Relation between binomial coefficients.** **Also be aware of alternative notations such as $\binom{n}{r}$ and ^nC_r** **Considered further in Paper 3 Section 4.1.**
		Extend to any rational n, including its use for approximation; be aware that the expansion is valid for $\left\lvert\frac{bx}{a}\right\rvert < 1$ (proof not required)	May be used with the expansion of rational functions by decomposition into partial fractions May be asked to comment on the range of validity.

The Binomial Series: Recap

Recall that if n is a positive integer

$$\boxed{(a+b)^n = a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + \cdots}$$

$$\boxed{(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + {}^nC_r\, x^n}$$

Also $\boxed{(a+b)^n = a^n\left(1+\frac{b}{a}\right)^n}$

Examples

1. Expand $(1+x)^{11}$ up to and including the term in x^3

2. Expand $(1-2x)^8$ up to and including the term in x^3

Binomial Expansion for Negative/ Fractional Powers

Example

1. Use the binomial expansion to find the first four terms of $\frac{1}{1+x}$

2. Use the binomial expansion to find the first four terms of $\sqrt{1-3x}$

An infinite expansion $(1+x)^n$ is valid if $|x|<1$

Quickfire Examples:

1. Expansion of $(1+2x)^{-1}$ valid if:

2. Expansion of $(1-x)^{-2}$ valid if:

3. Expansion of $\left(1+\frac{1}{4}x\right)^{\frac{1}{2}}$ valid if:

4. Expansion of $\left(1-\frac{2}{3}x\right)^{-1}$ valid if:

Combining Expansions

(*a*) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1+x+\frac{1}{2}x^2, \qquad |x|<1$$

(6)

Test Your Understanding

1. Find the binomial expansion of $\frac{1}{(1+4x)^2}$ up to an including the term in x^3.
State the values of x for which the expansion is valid.

2.

(*a*) Find the binomial expansion of

$$\sqrt{(1-8x)}, \qquad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3, simplifying each term.

(6)

(*b*) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1-8x)}$ is $\frac{\sqrt{23}}{5}$.

(2)

(*c*) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (*a*) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

(3)

Extension

[STEP I 2011 Q6] Use the binomial expansion to show that the coefficient of x^r in the expansion of $(1-x)^{-3}$ is $\frac{1}{2}(r+1)(r+2)$.

(i) Show that the coefficient of x^r in the expansion of $\frac{1-x+2x^2}{(1-x)^3}$ is r^2+1 and hence find the sum of the series

$$1+\frac{2}{2}+\frac{5}{4}+\frac{10}{8}+\frac{17}{16}+\frac{26}{32}+\frac{37}{64}+\cdots$$

(ii) Find the sum of the series

$$1+2+\frac{9}{4}+2+\frac{25}{16}+\frac{9}{8}+\frac{49}{64}$$

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