

U6 Chapter 4

Binomial Expansion

Chapter Overview

1. Binomial Series Recap
2. Binomial Expansion for negative/fractional powers
3. Constant is not 1: $(a + b)^n$
4. Using Partial Fractions

4 Sequences and series	4.1	<p>Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n; the notations $n!$ and ${}^n C_r$ link to binomial probabilities.</p> <p>Extend to any rational n, including its use for approximation; be aware that the expansion is valid for</p> $\left \frac{bx}{a} \right < 1 \text{ (proof not required)}$	<p>Use of Pascal's triangle.</p> <p>Relation between binomial coefficients.</p> <p>Also be aware of alternative notations such as $\binom{n}{r}$ and ${}^n C_r$</p> <p>Considered further in Paper 3 Section 4.1.</p> <p>May be used with the expansion of rational functions by decomposition into partial fractions</p> <p>May be asked to comment on the range of validity.</p>
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The Binomial Series: Recap

Recall that if n is a positive integer

$$(a + b)^n = a^n + {}^n C_1 a^{n-1}b + {}^n C_2 a^{n-2}b^2 + \dots$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + {}^n C_r x^n$$

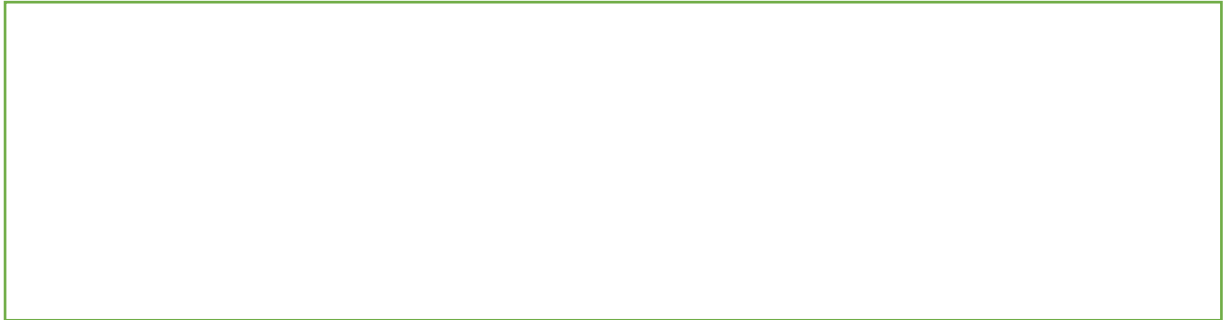
Also $(a + b)^n = a^n \left(1 + \frac{b}{a}\right)^n$

Examples

1. Expand $(1 + x)^{11}$ up to and including the term in x^3

2. Expand $(1 - 2x)^8$ up to and including the term in x^3

Binomial Expansion for Negative/ Fractional Powers



Example

1. Use the binomial expansion to find the first four terms of $\frac{1}{1+x}$

2. Use the binomial expansion to find the first four terms of $\sqrt{1-3x}$

An infinite expansion $(1 + x)^n$ is valid if $|x| < 1$

Quickfire Examples:

1. Expansion of $(1 + 2x)^{-1}$ valid if:

2. Expansion of $(1 - x)^{-2}$ valid if:

3. Expansion of $\left(1 + \frac{1}{4}x\right)^{\frac{1}{2}}$ valid if:

4. Expansion of $\left(1 - \frac{2}{3}x\right)^{-1}$ valid if:

Combining Expansions

(a) Use the binomial expansion to show that

$$\sqrt{\frac{1+x}{1-x}} \approx 1+x+\frac{1}{2}x^2, \quad |x| < 1$$

(6)

Test Your Understanding

1. Find the binomial expansion of $\frac{1}{(1+4x)^2}$ up to and including the term in x^3 .
State the values of x for which the expansion is valid.

2.

(a) Find the binomial expansion of

$$\sqrt[3]{(1 - 8x)}, \quad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3 , simplifying each term.

(6)

(b) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt[3]{(1 - 8x)}$ is $\frac{\sqrt[3]{23}}{5}$.

(2)

(c) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt[3]{23}$. Give your answer to 5 decimal places.

(3)

Extension

[STEP I 2011 Q6] Use the binomial expansion to show that the coefficient of x^r in the expansion of $(1 - x)^{-3}$ is $\frac{1}{2}(r + 1)(r + 2)$.

- (i) Show that the coefficient of x^r in the expansion of $\frac{1-x+2x^2}{(1-x)^3}$ is $r^2 + 1$ and hence find the sum of the series

$$1 + \frac{2}{2} + \frac{5}{4} + \frac{10}{8} + \frac{17}{16} + \frac{26}{32} + \frac{37}{64} + \dots$$

- (ii) Find the sum of the series

$$1 + 2 + \frac{9}{4} + 2 + \frac{25}{16} + \frac{9}{8} + \frac{49}{64}$$

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